

# Spectrum Sharing for Device-to-Device Communications in Cellular Networks: A Game Theoretic Approach

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**Abstract**—This paper studies the spectrum sharing problem between device-to-device (D2D) and cellular communications in a cellular network. In this network, the D2D links can access the spectrum controlled by a mobile network operator. Each D2D link can either access the sub-bands occupied by cellular subscribers or obtain a sub-band for its exclusive use. The D2D links with exclusive use of sub-bands can also share spectrum with each other. We observe that the above spectrum sharing problem is complex and there may not exist a stable spectrum sharing structure. We establish a hierarchical matching market with incomplete information to model and analyze the above D2D spectrum sharing problem. In our model, each D2D link is selfish and autonomous. We seek a Bayesian equilibrium of our market that achieves a stable spectrum sharing structure among all the D2D links. We derive a sufficient condition for which the Bayesian equilibrium exists. We propose a distributed algorithm which can detect whether this sufficient condition is satisfied and, if satisfied, achieve the Bayesian equilibrium. Our algorithm does not require each D2D link to know the payoffs of others and has the worst case complexity of  $O(L^3J)$  in each iteration where  $L$  is the number of D2D links,  $J$  is the number of cellular sub-bands.

**Index Terms**—Device-to-device communication, stable marriage, stable roommates, matching market, spectrum sharing, cellular network, game theory.

## I. INTRODUCTION

In the traditional cellular network structure, all traffic is forwarded and relayed by the network infrastructure (e.g., base station) even when the sources and destinations are close to each other. This not only increases communication delay and energy consumption but also reduces network reliability. For example, in cellular networks, failure of a base station (BS) could lead to mobile service outage for the entire coverage area of the corresponding cell. Device-to-device (D2D) communication without using the BS to forward the traffic provides an efficient method to increase the capacity and reliability of wireless networks.

Another issue is that the traditional exclusive spectrum ownership model has resulted in low spectrum utilization

efficiency for a significant portion of the time [1], [2]. One technique that promises to address this problem is spectrum sharing which allows the under-utilized spectrum to be shared by other devices.

Allowing both D2D communications and spectrum sharing in cellular networks can improve network capacity, reliability and spectrum utilization efficiency. However, D2D links are generally established autonomously and cannot be fully controlled by the BS. In addition, choosing the wrong spectrum sharing pair of D2D links and cellular subscribers can result in high cross-interference, which may adversely affect both D2D links and cellular subscribers.

This motivates the work in this paper, where we investigate the optimization of D2D spectrum sharing in a cellular network. We propose a general network model in which D2D links can access the spectrum licensed to an operator that provides coverage in the area of interest. Each D2D link can either share the sub-band currently occupied by cellular subscribers or apply for an empty sub-band for its exclusive use. The D2D links are autonomous and, to further increase spectrum utilization efficiency, the D2D links with exclusive use of a sub-band can also aggregate and share spectrum with each other without consulting the operator.

The distribution and autonomy of D2D links make game theory a natural tool to study and analyze D2D communication systems. In this paper, we establish a new game theoretic framework, called a hierarchical matching market with incomplete information, to analyze the D2D spectrum sharing problem. In our proposed game, each D2D link can share the spectrum with the existing cellular subscribers or apply for an exclusive sub-band. D2D links that are granted sub-bands for their exclusive use can also share spectrum among themselves. We seek the Bayesian equilibrium that achieves a stable spectrum sharing structure among D2D links. We observe that the Bayesian equilibrium of our proposed market may not always exist.

Fortunately, we can simplify the above problem by dividing the proposed hierarchical matching market into two sub-markets. More specifically, all D2D links will first compete for the spectrum controlled by the operator.

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If a D2D link wants to share the spectrum with existing cellular subscribers, it will enter the first sub-market, which is a two-sided one-to-one matching market with private belief, referred to as the sub-band allocation sub-market. If sharing spectrum currently used by subscribers cannot provide enough payoff for the D2D link or causes intolerably high interference to cellular subscribers, the D2D links will apply for sub-bands for exclusive use. To decide whether and with whom to share these sub-bands, those D2D links with exclusive sub-bands will enter the second sub-market, which is a one-sided one-to-one matching market with private belief, referred to as a D2D spectrum sharing sub-market. We derive a sufficient condition for the existence of a Bayesian equilibrium. We propose a distributed Bayesian belief updating algorithm that can detect whether this sufficient condition is satisfied and, if satisfied, approach a Bayesian equilibrium that can achieve a stable matching structure among all D2D links. We prove that all D2D links will stick to a stable matching structure once they reach this structure. Our proposed framework is general and the payoff of each D2D link can be any performance measure generated from its received signal-to-interference-noise ratio (SINR). In addition, each D2D link is not required to know the payoffs of others.

The rest of this paper is organized as follows. Related work is reviewed in Section II. The network model and problem formulation are presented in Section III. The hierarchical matching framework is proposed in Section IV. The numerical results are presented in Section V and the paper is concluded in Section VI.

## II. RELATED WORK

Most of the reported results on D2D communications have focused on resource allocation for one D2D source-to-destination link with specific performance goals. In [3], the authors applied power control and multi-hop routing discovery methods to improve the probability of outage for opportunistic D2D communications in a cellular network. The power control of D2D links in a cellular network has also been studied in [4]–[7]. In [8], the authors investigated the possible performance improvement brought by network coding and user cooperation in a D2D communication system. The authors in [9] have proposed a mechanism to support a D2D communication session in existing LTE cellular networks.

In this paper, we propose a hierarchical framework based on a matching market and find the stable structure in our proposed market. The two-sided stable matching problem has been widely studied from both theoretical and practical perspectives [10]–[15]. In this problem, each agent belonging to one side of the market has a preference about the agents belonging to the other side and tries to find a matching to optimize its performance. Many extensions of these problems have been studied in the literature. More specifically, the case of some agents on the one side only having preferences over a sub-set of

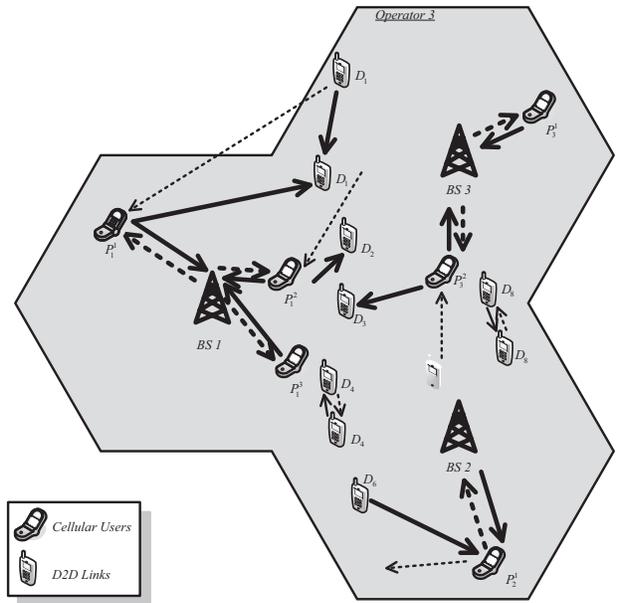


Fig. 1. Network model and sub-band sharing for a cellular network with D2D communications.

the agents on the other side was studied in [16]. The case that the agents from one side can have equal preference over multiple agents of the other side, called stable marriage with tie, has been studied in [17]. Empirical studies of the different variations of the stable marriage problem have been reported in [13], [18]. In most of these previous works, each player does not have any beliefs about the environment as well as the preference of others. In this paper, we allow each player to establish and maintain a private belief function. One work that is similar to our setting of private belief for agents is the belief-based coalition formation game proposed in [19]. However, that work assumes the belief functions are fixed and cannot be updated during the game, which is different from the setting of our paper, where we introduce a Bayesian belief update algorithm to allow each player to search for the Bayesian equilibrium solution that achieves a stable matching structure among all D2D links.

## III. NETWORK MODEL AND PROBLEM FORMULATION

### A. Network Model

Consider a network consisting of a set of  $L$  D2D links, labeled as  $\mathcal{D} = \{D_1, D_2, \dots, D_L\}$ . Each D2D link can access the spectrum of a cellular operator. The operator has been licensed a set  $\mathcal{J}$  of sub-bands which can be used to provide wireless service to a set of cellular subscribers, denoted as  $\mathcal{K}$ , through its infrastructure. We denote  $J = |\mathcal{J}|$ . Note that in cellular networks, the telecommunication service in each cell is provided and maintained by a BS. The BSs of the operator can exchange information through the core network (or, alternatively, via direct X2 links in LTE networks [20]) and hence can coordinate with each other to decide the sub-band allocations of the D2D links and cellular subscribers. To avoid cross interference among

TABLE I  
LIST OF NOTATIONS

Symbol	Definition
$\mathcal{J}$	Set of sub-bands
$\mathcal{K}$	Set of cellular subscribers
$\mathcal{D}$	Set of D2D links
$D_k$	the $k$ th D2D link
$P_l$	subscriber in sub-band $l$
$\varpi_{D_k}[l]$	Payoff of $D_k$ when accessing sub-band $l$
$\varpi_{D_k}[D_n]$	Payoff of D2D link $D_k$ when sharing a sub-band with $D_n$
$\bar{\varpi}_{D_k}$	Average payoff of D2D link $D_k$
$\eta_l[D_k]$	Revenue of the operator obtained from sub-band $l$
$\mathcal{R}_{D_k}^b$	Preference of $D_k$ over cellular sub-bands
$\mathcal{R}_{D_k}^d$	Preference of $D_k$ over other D2D links with exclusive use of sub-bands
$\mathcal{R}^b$	Preference profile of all D2D links
$\mathcal{S}$	D2D and cellular spectrum sharing structure
$b_{D_k}$	Belief function of $D_k$
$\mathcal{C}$	Set of D2D links with exclusive use of sub-bands
$\bar{\alpha}$	Labeling sequence of D2D links with exclusive use of sub-bands

subscribers, several frequency reuse technologies have been applied in existing cellular systems [21], [22]. Therefore, in this paper, we concentrate on a limited geographic area of interest that is covered by the operator, as depicted in Figure 1, and assume that there is no cross interference among subscribers. Each D2D link can only access one sub-band. Let the subscriber using sub-band  $l$  be  $P_l$  for  $P_l \neq 0$ . We use  $P_l = 0$  to denote that sub-band  $l$  is unoccupied by any cellular subscriber.

Note that each D2D link corresponds to a communication channel between a D2D source and its corresponding destination, and each cellular subscriber corresponds to a communication channel between a cellular subscriber and the corresponding BS. In this paper, we focus on spectrum sharing between D2D links and a cellular network. Specifically, each D2D link can share a full-sized cellular sub-band occupied by a cellular subscriber. If the spectrum sharing with a cellular subscriber cannot provide sufficient quality of service (QoS), each D2D link can also apply for a vacant sub-band for its exclusive use<sup>1</sup>. To further improve the spectrum utilization efficiency, the D2D links having exclusive use of sub-bands can also choose to share their sub-bands with each other.

In this paper, we assume that each sub-band can at most contain two users (either two D2D links or one D2D link and one cellular subscriber). This assumption is reasonable in practical system implementations because wireless channel gains generally change from time to time, and imposing a limit of two users to share one sub-band allows the operator to evaluate and control the cross interference between the users. For example, if either the D2D link or cellular subscriber, or both, observes higher-than-tolerable interference, the operator can remove the D2D link from the sub-band. If more than

<sup>1</sup>Because access to licensed spectrum is expensive, the exclusive sub-band given to each D2D link may, in practice, be narrower than the full-sized sub-band allocated to the cellular subscribers.

two users share the same sub-band (e.g., two D2D links share a sub-band with a cellular subscriber), it will be difficult to evaluate which user causes the highest interference to others, or which D2D link should be removed.

In our model, each D2D link will first apply for a cellular sub-band, which can either be unoccupied or occupied by a cellular subscriber. Let the payoff of  $D_k$  obtained by accessing sub-band  $l$  be  $\varpi_{D_k}[l]$ . In this paper, we consider a general model and the payoff of each D2D link can be any function of its received signal to interference plus noise ratio (SINR). For example, if the D2D link wants to maximize its transmit rate per bandwidth price, the payoff function of D2D link  $D_k$ , when it uses sub-band  $l$ , can be written as

$$\varpi_{D_k}[l] = \frac{\rho_{D_k}}{e(\rho_{D_k})} \log(1 + \text{SINR}_{D_k}[l]), \quad (1)$$

where  $\rho_{D_k}$  is the bandwidth occupied by D2D link  $D_k$  and  $e(\rho_{D_k})$  is the price of bandwidth paid to the network operator.  $\text{SINR}_{D_k}[l]$  is the signal to noise ratio received at  $D_k$ , given by

$$\text{SINR}_{D_k}[l] = \begin{cases} \frac{h_{D_k}[l]w_{D_k}}{1+h_{P_l D_k}w_{P_l}}, & \text{if } P_l \neq 0, \\ h_{D_k}[l]w_{D_k}, & \text{if } P_l = 0. \end{cases} \quad (2)$$

where  $h_{D_k}[l]$  is the channel gain between the source and destination of D2D link  $D_k$  in sub-band  $l$ ,  $h_{P_l D_k}$  is the channel gain between subscriber  $P_l$  and D2D link  $D_k$ , and  $w_{P_l}$  is the transmit power of subscriber  $P_l$  for  $P_l \neq 0$ . We have  $w_{P_l} = 0$  and  $h_{P_l D_k} = 0$  if  $P_l = 0$ .

If at least two D2D links have been allocated empty sub-bands for exclusive use, they can aggregate and share their sub-bands with each other by forming D2D spectrum sharing pairs. Let us consider a spectrum sharing pair formed by two D2D links  $D_k$  and  $D_n$ . We can write the payoff of D2D link  $D_k$  as  $\varpi_{D_k}[D_n]$ , for  $D_k \neq D_n$ . If two D2D links  $D_k$  and  $D_n$ ,  $k \neq n$ , with exclusive sub-bands share their spectrum with each other using random access [23] and agree to equally divide the prices they paid for spectrum, the payoff to each D2D link (e.g.,  $D_k$ ) is given by

$$\varpi_{D_k}[D_n] = \frac{2(\rho_{D_k} + \rho_{D_n})}{e(\rho_{D_k}) + e(\rho_{D_n})} \log(1 + \text{SINR}_{D_k}[D_n]), \quad (3)$$

where  $\text{SINR}_{D_k}[D_n] = \frac{h_{D_k}w_{D_k}}{1+h_{D_n D_k}w_{D_n}}$ ,  $h_{D_n D_k}$  is the channel gain between D2D links  $D_k$  and  $D_n$  and  $w_{D_n}$  is the transmit power of  $D_n$ <sup>2</sup>.

The revenue  $\eta_l[D_k]$  obtained by the operator from D2D link  $D_k$  that shares sub-band  $l$  can be a function of the resulting interference, i.e., we have

$$\eta_l[D_k] = \begin{cases} g(h_{D_k P_l}w_{D_k}), & P_l \neq 0, \\ g(h_{D_k}[l]w_{D_k}), & P_l = 0. \end{cases} \quad (4)$$

<sup>2</sup>The channel gain between  $D_k$  and  $D_n$  is actually the channel gain between the source of  $D_k$  and the destination of  $D_n$  and the transmit power of  $D_n$  is the transmit power of the source of  $D_n$ .

More specifically, if the price  $\eta_l[D_k]$  is a linear function of the received interference at  $P_l$ , we have  $g(h_{D_k P_l} w_{D_k}) = \beta h_{D_k P_l} w_{D_k}$  and  $g(h_{D_k[l]} w_{D_k}) = \beta h_{D_k[l]} w_{D_k}$  where  $\beta$  is the pricing coefficient of the operator [24]<sup>3</sup>.

The list of major notation used in this paper is provided in Table I.

### B. Problem Formulation

From the above description, we can observe that the D2D and cellular spectrum sharing system consists of a hierarchical structure. In this structure, all D2D links will first decide which cellular sub-bands they want to apply for, which we refer to as the *cellular sub-band allocation problem*. Those D2D links allocated empty sub-bands for exclusive use will then decide how to share their sub-bands with each other, which we refer to as the *D2D spectrum sharing problem*. We provide a more detailed description of these two problems below.

1) *Cellular Sub-band Allocation Problem*: To maintain an appropriate quality of service for each cellular subscriber, each operator needs to control the spectrum usage. If a D2D link  $D_k$  wants to access a sub-band, it needs to first send a request to the operator and only access the spectrum when its request has been approved. If more than one D2D link wants to access the same sub-band, the operator will need to reject the requests of some D2D links. In this paper, we assume the operator always wants to maximize its revenue in every sub-band. In other words, if the operator receives requests from more than one D2D link for the same sub-band, it will allocate this sub-band to the D2D link that can provide the highest revenue. Let us denote the preference of the operator about each sub-band  $l$  over all D2D links as  $\mathcal{R}_l$ . We also write  $\mathcal{R}^o = \{\mathcal{R}_l\}_{l \in \mathcal{J}}$ . In this paper, we mainly focus on the sub-band allocation and sharing between D2D links and hence assume  $\mathcal{R}^o$  is pre-decided by the operator and unrelated to the sub-band allocation of D2D links. If sharing a sub-band with a cellular subscriber cannot provide adequate performance to the D2D link, the cellular subscriber, or both, each of these D2D links will then be allocated an exclusive sub-band, which can either be used by itself or shared with other D2D links.

In this paper, we assume each D2D link cannot predict the sub-bands requested by other D2D links or the decision process of the operator about the cellular sub-band usage. In other words, each D2D link is uncertain about their final allocated sub-bands when it submits its sub-band request to the operator. We refer to this as *sub-band allocation uncertainty*. For example, if more than one D2D link have the same preferred sub-band  $l$ , only one of them can eventually get sub-band  $l$  and requests from all the other D2D links will be

<sup>3</sup>Note that this revenue function has been used in many previously reported results [24], [25]. One advantage of this revenue function is that the operator can adjust the pricing coefficient to control the interference caused by the D2D links (the higher the prices, the fewer D2D links who can afford the price and hence less interference from the D2D links to the cellular network).

rejected. These rejected D2D links will then send requests for other sub-bands according to their preference. The process will continue until all the D2D links have been allocated sub-bands. In this case, the strategy of each player cannot simply be its most preferred sub-band but should be a vector of all sub-bands in order of preference. For example, consider a network with two sub-bands labeled as 1 and 2. If a player  $D_k$  believes that accessing sub-band 2 could provide a higher payoff than sub-band 1, the preference of player  $D_k$  is given by  $\mathcal{R}_{D_k}^b = \langle 2, 1 \rangle$ . We use  $\tilde{r}$  to denote the labels of the sequence order of the preference, i.e., we can rank all the sub-bands from the most to least preferred ones for D2D link  $D_k$  and write its preference as  $\mathcal{R}_{D_k}^b = \langle \tilde{r}_{D_k}^1, \tilde{r}_{D_k}^2 \rangle$ , where  $\tilde{r}_{D_k}^1 = 2$  and  $\tilde{r}_{D_k}^2 = 1$  in this example. We write  $\mathcal{R}^b = \{\mathcal{R}_{D_k}^b\}_{D_k \in \mathcal{D}}$ .

2) *D2D Spectrum Sharing Problem*: Because, after the operator allocates the available sub-bands, the D2D links with exclusive use of a sub-band will not be fully controlled by the operator<sup>4</sup>, each D2D link can make an autonomous decision about its spectrum sharing partner in the network without notifying the operator. If at least two D2D links have been allocated sub-bands for exclusive use, they can aggregate and share their allocated sub-bands with each other. In this case, these D2D link will then need to decide whether to share their sub-bands with each other. Following the same line as the cellular sub-band allocation problem, each D2D link  $D_k$  will then need to submit its spectrum sharing request to another D2D link  $D_n$  with an exclusive sub-band. A D2D spectrum sharing pair can only be formed if the request of  $D_k$  has been approved by  $D_n$ . Let us denote the preference of each D2D link  $D_k$  over other D2D links with exclusive use of sub-bands as  $\mathcal{R}_{D_k}^d$ . We also use  $\tilde{v}_{D_k}$  to denote the sequence order of  $D_k$ 's preference about the other D2D links.

The sub-band allocation uncertainty in the cellular sub-band allocation problem directly leads to the second uncertainty for D2D links, which we refer to as *D2D spectrum sharing uncertainty*, when we allow two D2D links with exclusive sub-bands to aggregate their spectrum. More specifically, in our model, each D2D link will first decide whether to share the sub-bands currently occupied by the cellular subscribers or apply for an empty sub-band for exclusive use. If a D2D link believes sharing a sub-band with a cellular subscriber can provide a higher payoff than obtaining an empty sub-band for exclusive use, it still does not know about whether obtaining an empty sub-band first and sharing with other D2D links later can further improve its payoff.

### C. Game Modeling

In this paper, we assume each D2D link is rational and wants to maximize its payoff. We model the spectrum sharing problem between D2D links and the cellular

<sup>4</sup>Since the D2D links using vacant sub-bands cannot cause any interference to the cellular network, it is not necessary for the operator to monitor or control the interference and spectrum usage of these D2D links.

network as a hierarchical matching market with incomplete information, which is defined as follows.

*Definition 1:* We define the D2D and cellular spectrum sharing game as a hierarchical matching market with incomplete information given by  $\langle \mathcal{D}, \mathcal{J}, \mathcal{T}, \succ \rangle$  where  $\mathcal{D}$  is the set of D2D links,  $\mathcal{J}$  is the set of cellular sub-bands,  $\mathcal{T}$  is the set of types, and  $\succ$  is the preference of each D2D link (or operator) over all the cellular sub-bands and other D2D links (or all D2D links).

The above game can be regarded as the generalization of a hedonic coalition formation game and one-sided and two-sided stable matching markets [10], [11], [26] in the sense that we allow some D2D links to match (or form coalitions) with other D2D links, while other D2D links can match (or form coalitions) with cellular sub-bands.

Each D2D link  $D_k$  can maintain a private belief, denoted as  $b_{D_k}$ , about the allocated sub-band preferences of other D2D links given its own preference, i.e., we have

$$b_{D_k} \left( \mathcal{R}_{-D_k}^b, \mathcal{R}_{D_k}^b \right) = \Pr \left( \mathcal{R}_{-D_k}^b | \mathcal{R}_{D_k}^b \right). \quad (5)$$

where we use  $-D_k$  to denote the set of all D2D links except  $D_k$ .

In this paper, we mainly focus on the spectrum sharing decision process of D2D links. Let us define a D2D spectrum sharing structure as follows.

*Definition 2:* We define the D2D and cellular spectrum sharing structure  $\mathcal{S}$  as a function from the set  $\mathcal{D}$  to  $\mathcal{D} \cup \mathcal{J}$  such that  $\mathcal{S}(D_k) \in \mathcal{D} \cup \mathcal{J} \cup \emptyset$ ,  $\mathcal{S}(l) \in \mathcal{D} \cup \emptyset$ ,  $\mathcal{S}(D_k) = l \Leftrightarrow \mathcal{S}(l) = D_k$  and  $\mathcal{S}(D_k) = D_n \Leftrightarrow \mathcal{S}(D_n) = D_k$  for every  $D_k, D_n \in \mathcal{D}$  and  $l \in \mathcal{J}$ .

We can have the average payoff of each D2D link  $D_k$  as follows,

$$\begin{aligned} & \bar{\omega}_{D_k} \left( b_{D_k}, \mathcal{R}^b \right) \\ &= \sum_{\mathcal{R}_{-D_k}^b \in \mathbb{P}^{L-1}} b_{D_k} \left( \mathcal{R}_{-D_k}^b | \mathcal{R}_{D_k}^b \right) \\ & \left( \Pr \left( \mathcal{S}(D_k) = l | \mathcal{R}^b \right) \bar{\omega}_{D_k} [l] \right. \\ & \left. + \Pr \left( \mathcal{S}(D_k) = D_n | \mathcal{R}^b \right) \bar{\omega}_{D_k} [D_n] \right) \end{aligned} \quad (6)$$

where  $\mathbb{P}^{L-1}$  is the set of all the possible sub-band preferences of  $L-1$  D2D links.

Note that, if we can introduce a mechanism such that, for a given preference profile  $\mathcal{R}^b$ , both the sub-band allocation and the spectrum sharing structure will be uniquely determined, we have

$$\begin{aligned} \Pr \left( \mathcal{S}(D_k) = l | \mathcal{R}^b \right) &= \begin{cases} 1, & \text{if } \mathcal{S}(D_k) = l, \\ 0, & \text{Otherwise.} \end{cases} \quad (7) \\ \Pr \left( \mathcal{S}(D_k) = D_n | \mathcal{R}^b \right) &= \begin{cases} 1, & \text{if } \mathcal{S}(D_k) = D_n, \\ 0, & \text{Otherwise.} \end{cases} \quad (8) \end{aligned}$$

We seek a Bayesian equilibrium of our proposed game which can achieve a stable D2D and cellular spectrum sharing structure. We present the formal definition of Bayesian equilibrium as follows.

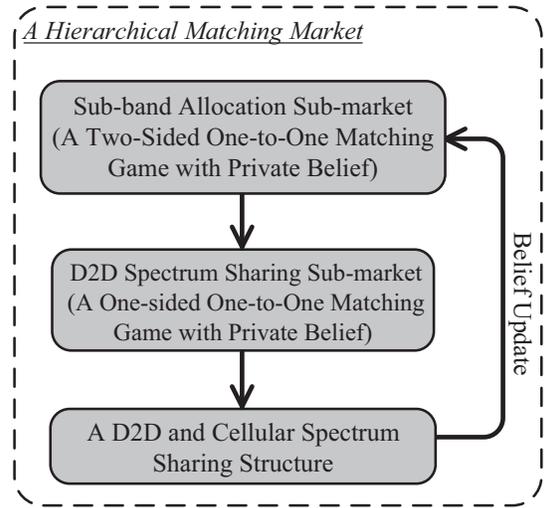


Fig. 2. A hierarchical matching framework for the D2D spectrum sharing game.

*Definition 3:* A Bayesian equilibrium of D2D and cellular spectrum sharing game is a preference (strategy) profile  $\mathcal{R}^{b*}$  such that

$$\bar{\omega}_{D_k} \left( b_{D_k}, \mathcal{R}_{D_k}^{b*}, \mathcal{R}_{-D_k}^{b*} \right) \geq \bar{\omega}_{D_k} \left( b_{D_k}, \mathcal{R}_{D_k}^b, \mathcal{R}_{-D_k}^{b*} \right) \quad \forall \mathcal{R}_{D_k}^b \in \mathbb{P}_J \text{ and } D_k \in \mathcal{D} \quad (9)$$

where  $\mathbb{P}_J$  is the set of permutations of the sequence of  $J$  sub-bands of the operator.

As we will show later, the Bayesian equilibrium may not always exist. Therefore, it is important to first derive a sufficient condition for which a Bayesian equilibrium exists and then propose a distributed algorithm to approach this equilibrium solution.

#### IV. A HIERARCHICAL MATCHING FRAMEWORK FOR SPECTRUM SHARING GAME

As observed in Section III-B, the D2D and cellular spectrum sharing problem has a hierarchical structure. More specifically, the proposed hierarchical matching market can be further divided into two sub-markets. All D2D links will first decide their cellular sub-bands by playing the *sub-band allocation sub-market* (to be discussed in Section IV-A). The D2D links that have been assigned a sub-band for their exclusive use can then play the *D2D spectrum sharing sub-market* with the objective of pairing up D2D links that can benefit from aggregating and sharing their sub-bands (to be discussed in Section IV-B). Finally, in Section IV-C, we derive a sufficient condition for which the Bayesian equilibrium exists. We also introduce a belief updating algorithm to detect whether this condition is satisfied and, if satisfied, approach the Bayesian equilibrium that achieves a stable matching structure among all D2D links. The relationship between the different sub-markets and the Bayesian belief updating method is described in Figure 2.

### A. Sub-band Allocation Sub-market

Each D2D link can either choose to share the spectrum of cellular subscribers or to apply for a vacant sub-band for exclusive use. We can model this as a two-sided matching market with private belief. The two-sided one-to-one matching market is also known as the *stable marriage market*. In this market, a set of D2D links proposes to a set of cellular subscribers (controlled by the operator) according to their ordered preference and the operator can then decide whether or not to accept the proposal from each D2D link according to its preference order.

We formally define the sub-band allocation sub-market as follows:

*Definition 4:* Let us define the (*cellular*) *sub-band allocation sub-market* as a two-sided matching market with private belief  $\mathcal{G} = \langle \mathcal{D}, \mathcal{J}, \mathbf{b}, \succ \rangle$  which consists of a set  $\mathcal{D}$  of D2D links, a set  $\mathcal{J}$  of cellular sub-bands,  $\mathbf{b} = \{b_{D_k}\}_{D_k \in \mathcal{D}}$ , the belief function of the D2D links, and the preference  $\succ$  of each D2D link (or cellular subscriber<sup>5</sup>) over the cellular subscribers (or D2D links).

We abuse the notation and use  $l \succ_{D_k} m$  to denote that D2D link  $D_k$  prefers sharing sub-band  $l$  over sub-band  $m$ . Similarly,  $D_k \succ_l D_n$  means the operator prefers to let D2D link  $D_k$  (as opposed to  $D_n$ ) occupy sub-band  $l$ . We use  $\mathcal{R}_{D_k}^b$  (or  $\mathcal{R}_l$ ) to denote the preference of D2D link  $D_k$  (or the operator) over all the cellular sub-bands (or all the D2D links). Let us define a matching between D2D links and cellular sub-bands of the operator as follows.

*Definition 5:* A (two-sided one-to-one) *matching*  $M$  between D2D links and cellular sub-bands is a function from the set  $\mathcal{D}$  of D2D links to the set  $\mathcal{J}$  of cellular sub-bands such that  $M(D_k) \in \mathcal{J}$ ,  $M(l) \in \mathcal{D}$  and  $M(D_k) = l \Leftrightarrow M(l) = D_k$  for every  $l \in \mathcal{J}$  and  $D_k \in \mathcal{D}$ .

An important concept in matching theory is stability, which is defined as follows.

*Definition 6:* A matching  $M$  is said to be *stable* if it cannot be strictly improved upon by any player or pair.

In a cellular network, it is the operator that controls the allocation of the licensed spectrum. Every time a D2D link wants to access the sub-band of a cellular subscriber, it consults the operator. The operator then evaluates the possible payoff in every cellular sub-band requested by the D2D links and establishes its preference for each sub-band over all the requesting D2D links. Using this preference, the operator can finally decide whether or not to approve the request of each D2D link.

We present the sub-band allocation algorithm below.

---

#### Algorithm 1: A Sub-band Allocation Algorithm

---

<sup>5</sup>As observed in Section III, to maintain the QoS of the existing cellular subscribers, the sub-band sharing between D2D links and cellular subscribers needs to be strictly controlled by the operator. Therefore, the preference of each cellular subscriber over the D2D links has to be established and maintained by the operator. To simplify our discussion, in this paper, we use the term ‘‘preference of each cellular subscriber’’ to denote the preference of the operator over the spectrum sharing between the D2D links and each of its cellular sub-bands.

---

*Input:* a set of belief functions  $\mathbf{b}$  for D2D links and a preference profile  $\mathcal{R}^o$  for the operator.

*Output:* a matching between D2D links and cellular sub-bands.

1) Each D2D link  $D_k$  decides its preference by

$$\mathcal{R}_{D_k}^b = \arg \max_{\mathcal{R}_{D_k}^b \in \mathbb{P}^{\mathcal{J}}} \varpi_{D_k}(b_{D_k}, \mathcal{R}_{D_k}^b), \quad (10)$$

2) WHILE at least one D2D link has not been allocated a sub-band,

- a) If  $\exists D_k$  and  $D_n$  such that  $\tilde{r}_{D_k}^1 = \tilde{r}_{D_n}^1$  and  $D_k \succ_{\tilde{r}_{D_k}^1} D_n$  for  $D_n \neq D_k$  and  $D_n, D_k \in \mathcal{D}$ , operator  $i$  rejects  $D_n$ ,
- b) If the request made by a D2D link  $D_k$  has been rejected,  $D_k$  removes  $\tilde{r}_{D_k}^1$  from its preference list and updates  $\tilde{r}_{D_k}^i = \tilde{r}_{D_k}^{i+1}$  for all  $i \in \{1, 2, \dots, |\mathcal{R}_{D_k}^b| - 1\}$ .  $D_k$  then submits a request message for  $\tilde{r}_{D_k}^1$  in the updated  $\mathcal{R}_{D_k}^b$ .

ENDWHILE

---

We have the following results for Algorithm 1.

*Proposition 1:* Suppose the belief of every D2D link is fixed. There always exists a stable matching for the sub-band allocation sub-market. Algorithm 1 generates a unique and stable matching.

*Proof:* From Step 2-b) in Algorithm 1, we can easily show that if the request from a D2D link  $D_k$  for a sub-band  $l$  has been rejected, there must exist at least another D2D link which is strictly preferred for sub-band  $l$  over  $D_k$ , and hence any matching between D2D link  $D_k$  and sub-band  $l$  must not be stable. Using this observation, we can also establish that if a D2D link  $D_k$  has been rejected for sub-band  $l$ , all the D2D links that are less preferable for sub-band  $l$  than  $D_k$  will also be rejected for sub-band  $l$ . Combining the above two observations, if D2D link  $D_k$  and sub-band  $l$  are matched at the end of Algorithm 1, we can claim that there is no other D2D link that is more preferred for sub-band  $l$  than the  $D_k$ . This is from the fact that if such a D2D link, say  $D_n$ , exists, the request of D2D link  $D_k$  for sub-band  $l$  will be rejected according to Step 2-a) of Algorithm 1. And each D2D link  $D_k$  matched to sub-band  $l$  cannot find another sub-band that is more preferable than sub-band  $l$  in the stable matching because, if such a sub-band exists, these D2D links will not send a request message for sub-band  $l$ . This concludes our proof. ■

From Proposition 1, we can observe that if the preference of each D2D link is fixed, then the resulting matching as well as the set of D2D links with exclusive use of sub-bands will also be fixed. Let the set of D2D links allocated sub-bands for exclusive use after Algorithm 1 be  $\mathcal{C}$ . Note that according to Proposition 1,  $\mathcal{C}$  is a function of  $\mathcal{R}^b$ , i.e.,  $\mathcal{C}(\mathcal{R}^b)$ . To simplify our notation, we write  $\mathcal{C}(\mathcal{R}^b)$  as  $\mathcal{C}$ .

We have the following result about the complexity of Algorithm 1.

*Proposition 2:* The complexity of Algorithm 1 in the worst case is  $O(LJ)$ <sup>6</sup>.

*Proof:* From Algorithm 1, the worst case happens when all D2D links can only be allocated with their least preferred sub-bands, after receiving  $(J - 1)$  rejections from the operator. In this worst case, every D2D link will first send requests to the  $J - 1$  most preferred sub-bands and receive rejections for all of them. In this case, the number of requests sent by all D2D links is given by  $L(J - 1)$ , which results in complexity of  $O(LJ)$ . This concludes the proof. ■

### B. D2D Spectrum Sharing sub-market

If sharing sub-bands with cellular subscribers cannot provide adequate payoff to some D2D links (e.g., some D2D links are closely located to some cellular subscribers and spectrum sharing causes intolerable cross interference), they will be given exclusive use of a sub-band and decide whether or not to share the sub-band with other D2D links. In this subsection, we consider the spectrum sharing between D2D links with exclusive sub-bands, after the sub-band allocation process in the previous section. In this case, the sub-market will no longer be a two-sided matching market, because each D2D link can find a match with any other D2D link with exclusive use of a sub-band in the entire network. We can then solve the problem by proposing a one-sided one-to-one matching market with private belief. The one-sided one-to-one matching market is also known as the *stable roommate market*, in which each student (or, in our model, D2D link) will choose another student (or, in our model, another D2D link) to share the same dormitory (in our model, a sub-band).

Let us now define the D2D spectrum sharing sub-market as follows.

*Definition 7:* We define the D2D spectrum sharing sub-market as a one-sided one-to-one matching market with private belief  $\mathcal{G} = \langle \mathcal{D}, \mathcal{J}, \mathbf{b}, \succ \rangle$  where  $\mathbf{b}$  is the belief function, and  $\succ$  is the preference of each D2D link over other D2D links with exclusive sub-bands. We abuse the notation and use  $D_m \succ_{D_n} D_k$  to denote that  $D_n$  prefers  $D_m$  to  $D_k$ .

*Definition 8:* A (one-sided one-to-one) *matching*  $M'$  between two D2D links is a function from the sets  $\mathcal{C}$  to  $\mathcal{C}$  such that  $M'(D_k) \in \mathcal{C} \cup \emptyset$ ,  $M'(D_n) \in \mathcal{C} \cup \emptyset$ , and  $M'(D_k) = D_n \Leftrightarrow M'(D_n) = D_k$  for every  $D_n, D_k \in \mathcal{C}$ .

Let us now discuss how to establish the preference for each D2D link when spectrum sharing between two D2D links is allowed in the cellular network. In this case, each D2D link will need to evaluate and rank its resulting payoffs when sharing a sub-band with another D2D link that also has exclusive access to a sub-band. One way to achieve this is for all D2D links with exclusive sub-bands to sequentially broadcast a training message to allow all

D2D links to estimate their resulting payoff when sharing their sub-bands with each other.

Different from the two-sided matching markets discussed in the previous subsection, in a one-sided matching market, there may not always exist a stable matching. One of the main reasons for this is the possible existence of *rotations* in the resulting preferences.

*Definition 9:* A *rotation* for a sequence of D2D link preferences is a sequence of D2D links  $(\hat{D}_0, \hat{D}'_0), (\hat{D}_1, \hat{D}'_1), \dots, (\hat{D}_{k-1}, \hat{D}'_{k-1})$  such that  $\hat{D}_i \neq \hat{D}_j$  for  $i \neq j$  and  $\hat{D}_i, \hat{D}_j \in \mathcal{C}$ , and  $\hat{D}'_i$  is the most preferred D2D link for  $\hat{D}_i$  and  $\hat{D}'_{i+1}$  is the second most preferred D2D link for  $\hat{D}_i$  for all  $i \in \{1, 2, \dots, k\}$  where the subscripts are taken modulo  $k$ .

We need to find a way to remove the rotations from the possible matching structures. As observed in [11], [12], [17], [27], removing rotations with different sequences may result in different matching<sup>7</sup>. This problem can be solved by taking advantage of the labeled identity of each D2D link. More specifically, in a D2D communication network, each D2D link has a specific commonly known identification number, referred to as a *label*, that is used by other D2D links to recognize it. We can then order all D2D links with exclusive sub-bands according to a fixed sequence of their labels, i.e., we denote the ordered D2D links as  $\tilde{a}_i$  and the vector of all the D2D links in  $\mathcal{C}$  can be denoted as  $\tilde{\mathbf{a}} = \langle \tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_{|\mathcal{C}|} \rangle$  for  $\tilde{a}_i \in \mathcal{C}$ .

Removing the rotations also requires communication among D2D links with exclusive sub-bands. More specifically, each D2D link will sequentially send a rotation detection signal to each other to see if a rotation like sequence can be detected [11], [12], [17], [27]. If a rotation has been detected, all D2D links in the sequence of rotation will remove the rotation from their preference list.

Let us now present the detailed algorithm below.

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#### Algorithm 2: A D2D spectrum sharing Algorithm

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*Input:* a preference profile  $\mathcal{R}^d = \{\mathcal{R}_{D_k}^d\}_{D_k \in \mathcal{C}}$  for D2D links with exclusive use of sub-bands, and a set of ordered sequences  $\tilde{\mathbf{a}}$ .

*Output:* If a stable matching exists, output the stable matching. Otherwise, report the non-existence of stable matchings.

*Initialization:*

- 1) WHILE at least one D2D link does not receive any request or there exists at least one D2D link  $D_k \in \mathcal{C}$  such that  $\mathcal{R}_{D_k}^d = \emptyset$ 
  - a) Each D2D link  $D_k$  sends the request to its most preferred D2D link in  $\mathcal{R}_{D_k}^d$  (e.g.,  $\tilde{v}_{D_k}^1 = D_n$ ). If  $\exists D_k$  and  $D_n$  such that  $\tilde{v}_{D_k}^1 = \tilde{v}_{D_n}^1$  and  $D_k \succ_{\tilde{v}_{D_k}^1} D_n$  for  $D_n \neq D_k$  and  $D_n, D_k \in \mathcal{C}$ ,  $\tilde{v}_{D_k}^1$  rejects  $D_n$ .

<sup>7</sup>It has been proved in [11] that a stable matching is associated with a unique set of rotations. Therefore, if the rotation detection and removal sequence can be uniquely decided, the set of observable rotations as well as the stable matching will also be fixed.

<sup>6</sup>In this paper, we follow Bachmann-Landau notations:  $f = O(g)$  if  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < +\infty$ .

- b) If the request made by a D2D link  $D_k$  has been rejected, D2D link  $D_k$  removes  $\tilde{r}_{D_k}^1$  from its preference list and updates  $\tilde{r}_{D_k}^i = \tilde{r}_{D_k}^{i+1}$  for all  $i \in \{1, 2, \dots, |\mathcal{R}_{D_k}^d| - 1\}$ . D2D link  $D_k$  then submits a request message to D2D link  $\tilde{r}_{D_k}^1$  in the updated  $\mathcal{R}_{D_k}^d$ .
- c) Whenever a D2D link  $D_k$  receives the request from another D2D link  $D_m$ ,  $D_k$  removes all the D2D links that are less preferable than  $D_m$  from  $\mathcal{R}_{D_k}^d$ ,

ENDWHILE

- 2) WHILE  $\exists D_k \in \mathcal{C}$ ,  $|\mathcal{R}_{D_k}^d| \geq 2$ ,
- a) For every  $D_k$  with  $|\mathcal{R}_{D_k}^d| \geq 2$ , it detects whether a rotation sequence exists in their preference list according to the labeling sequence  $\tilde{a}$ .
- b) Once a rotation has been detected, then the D2D links in the sequence of rotation remove the rotation from their preference list.

ENDWHILE

- 3) IF  $\exists D_k \in \mathcal{C}$ ,  $\mathcal{R}_{D_k}^d = \emptyset$  then there is no stable matching.  
 ELSE a stable matching is achieved by  $M(D_k) = \tilde{v}_{D_k}^1, \forall D_k \in \mathcal{C}$ .  
 ENDIF

If none of the preference lists of the D2D links are empty after removing the rotations in the above algorithm, the resulting matching structure is stable. We have the following results.

*Proposition 3:* Suppose the set  $\mathcal{C}$  of D2D links being allocated empty sub-bands for exclusive use are fixed. Algorithm 2 either reports no stable matching exists or generates a stable matching structure.

*Proof:* In Step 1) of Algorithm 2, the D2D links with exclusive use of sub-bands sequentially detect and remove the rotations from their preferences. We can use Theorem 4.2.1 in [11] to prove that if a matching exists after removing all the rotations, this matching is stable for all the D2D links with exclusive use of a sub-band. This concludes the proof. ■

From the above proposition, if Algorithm 2 reports a stable matching structure, we can claim that at least one stable matching structure exists. This can be regarded as a sufficient condition for the existence of a stable matching for the D2D spectrum sharing sub-market.

We have the following results about the complexity of the above algorithm.

*Proposition 4:* The complexity of Algorithm 2 in the worst-case is given by  $O(|\mathcal{C}|^2)$ .

*Proof:* In the worst case of Algorithm 2, all D2D links with exclusive use of a sub-band will send their requests to each other before they find their preferred partners. This requires all D2D links in  $\mathcal{C}$  to send  $|\mathcal{C}|^2$  requests. ■

In this and previous subsections, we have proposed two algorithms to achieve a stable matching for sub-band allocation and D2D spectrum sharing sub-markets. These two algorithms can be combined to achieve a stable hierarchical matching structure for the D2D and cellular

spectrum sharing game proposed in Section III-B. More specifically, we can search for a D2D and cellular spectrum sharing structure by allowing all D2D links to first use Algorithm 1 to search for a unique and stable matching for the sub-band allocation sub-market and all those D2D links matched with empty sub-bands can then use Algorithm 2 to search for their stable matching for the D2D spectrum sharing sub-market.

### C. A Belief Updating Algorithm

It is observed in Propositions 1 that if the belief of every D2D link is fixed, then the sub-band allocation scheme generated by Algorithm 1 will be fixed too. According to Proposition 3, the resulting matching between D2D links with exclusive sub-bands generated by Algorithm 2 will also be fixed. Therefore, it is important for each D2D link to estimate the true preference order that will maximize its payoff. However, to predict such preferences requires each D2D link to know the instantaneous payoffs and preferences of all the other D2D links which is generally impractical.

In this subsection, we introduce a Bayesian belief updating algorithm for all D2D links to iteratively update their beliefs. In our model, each D2D link can eavesdrop the preference order  $\mathcal{R}_{-D_k}^b$  previously submitted by all other D2D links to the operator after every matching process. We assume each D2D link is myopic and can then use the following equation to calculate its belief regarding the preferences of other D2D links in each iteration  $t$ ,

$$b_{D_k} \left( \mathcal{R}_{-D_k}^b [t] \right) = \frac{\theta_{D_k} \left( \mathcal{R}_{-D_k}^b [t] \right)}{t}, \quad (11)$$

where  $\theta_{D_k} \left( \mathcal{R}_{-D_k}^b [t] \right) = \sum_{u \in \{1, \dots, t\}} \text{Dir} \left( \mathcal{R}_{-D_k}^b [u] \right)$  is the number of times that all D2D links have their preference list as  $\mathcal{R}_{-D_k}^b$  in the first  $t$  time slots, and  $\text{Dir}(\cdot)$  is the Dirac delta function. After updating their beliefs, each D2D link uses equation (10) to decide its preference.

Let us present the belief updating algorithm as follows.

#### Algorithm 3: A Belief Updating Algorithm

WHILE the matching structure of D2D links is not stable,

- 1) Every D2D link establishes its preference and then selects the sub-band it wants to access using Algorithm 1.
- 2) After choosing their sub-bands, the D2D links with exclusive sub-bands decide whether to share their sub-bands with each other using Algorithm 2.
- 3) After all D2D links choose their sub-bands, they use equation (11) to update their beliefs and then use equation (10) to choose their preferences.

ENDWHILE

We now show that the result in Proposition 3 also holds if all the D2D links use the belief updating algorithm in (10). We have the following result about Algorithm 3.

**Theorem 1:** Suppose the beliefs of D2D links converge and there exists a stable matching in Step 2) of Algorithm 3 for every resulting belief function of D2D links. Then we have the following results:

- 1) The resulting preference profile  $\mathcal{R}^b$  is the Bayesian equilibrium of our proposed game described in Section III-C. For the resulting belief function at each D2D link, the resulting matching structure of D2D links is stable.
- 2) Suppose, in some time slot  $t$ , the D2D and cellular spectrum sharing structure  $\mathcal{S}[t]$  satisfies  $\mathcal{S}[t] = \mathcal{S}^*$ , where  $\mathcal{S}^*$  is the stable D2D and cellular spectrum sharing structure with the resulting beliefs. Then,  $\mathcal{S}[\tau] = \mathcal{S}^*$  for all  $\tau > t$  using Algorithm 3.

*Proof:* First, let us consider the first result. It can be easily observed that if the belief function of every D2D link  $D_k$  converges, all D2D links can establish their preferences and use Algorithm 1 to obtain a stable matching between D2D links and cellular sub-bands. D2D links can then use Algorithm 2 to generate the stable matching structure among D2D links with exclusive sub-bands. In other words, the matching structure resulting from both sub-markets is stable and deterministic for every given belief function of D2D links. Therefore, the hierarchical matching structure resulted from Algorithm 3 is stable.

Let us now consider the second result. If  $\mathcal{S}[t] = \mathcal{S}^*$  in time slot  $t$ , we then have  $\bar{\omega}_{D_k}(\mathcal{S}^*) > \bar{\omega}_{D_k}(\mathcal{S}')$ . Let us show that in the next time slot  $t + 1$ , each D2D link will stick with  $\mathcal{S}^*$  and will not change to other preferences. Let  $\mathcal{R}^{b*}$  be the preference profile that generates the D2D and cellular spectrum sharing structure  $\mathcal{S}^*$ . In time slot  $t + 1$ , D2D link  $D_k$  will update its belief as follows:

$$b_{D_k}(\mathcal{R}_{-D_k}^b, \mathcal{R}_{D_k}^b)[t + 1] = \alpha b_{D_k}(\mathcal{R}_{-D_k}^b, \mathcal{R}_{D_k}^b)[t] + (1 - \alpha)\mathbf{1}(\mathcal{R}_{D_k}^b[t + 1] = \mathcal{R}_{D_k}^{b*}) \quad (12)$$

where  $\mathbf{1}(\cdot)$  is the indicator function and  $\alpha = \frac{t}{t+1}$ . We then can rewrite the updated payoff function of  $D_k$  as

$$\bar{\omega}_{D_k}[t + 1] = \alpha \bar{\omega}_{D_k}(\mathcal{R}_{D_k}^b[t], \mathcal{R}_{-D_k}^b[t], b_{D_k}[t]) + (1 - \alpha) \bar{\omega}_{D_k}(\mathcal{R}_{D_k}^b[t + 1], \mathcal{R}_{-D_k}^b[t + 1], b_{D_k}[t + 1]), \quad (13)$$

which is a linear combination of  $\bar{\omega}_{D_k}[t]$  and  $\bar{\omega}_{D_k}[t + 1]$ . It can be easily observed that the resulting D2D and cellular spectrum sharing structure  $\mathcal{S}[t + 1] = \mathcal{S}[t] = \mathcal{S}^*$  maximizes both payoff functions on the right hand side of (13) and hence each D2D link  $D_k$  will have no intention to unilaterally change to another preference during the following time slots. This process will be repeated for the rest of the time slots. ■

Following the same line as Propositions 2 and 4, we have the following complexity result about Algorithm 3.

*Proposition 5:* The complexity of each iteration of Algorithm 3 is given by  $O(L^3J)$ .

*Proof:* Following the same line of Proposition 2 and 4, we can prove that the worst case complexity of the combined algorithm of Algorithms 1 and 2 is given by

$O(|\mathcal{C}|^2LJ)$ . If all  $L$  D2D links have been allocated empty sub-bands for exclusive use in Algorithm 1, i.e.,  $|\mathcal{C}| = L$ , the complexity of each iteration of Algorithm 3 is given by  $O(L^3J)$ . This concludes the proof. ■

## V. NUMERICAL RESULTS

In Section IV, we propose a hierarchical matching market consisting of two sub-markets and a belief updating algorithm to find the spectrum sharing structure between a set of D2D links and a cellular network. Our proposed framework is general in the sense that each of these proposed sub-markets can be individually applied to optimize each specific problem. For example, if D2D links with exclusive sub-bands cannot coordinate and establish preferences for sharing sub-bands among themselves, each D2D link can still choose a cellular sub-band using the sub-band allocation algorithm (Algorithm 1). In this section, we present numerical results to illustrate the performance of our proposed algorithm under different conditions. We compare the following three D2D spectrum sharing methods:

- 1) *Random Pairing:* each D2D link  $D_k$  randomly chooses a cellular sub-band  $l$ . If  $l$  is occupied by a subscriber  $P_l$ , a spectrum sharing pair can only be formed when the resulting payoffs for both  $D_k$  and  $P_l$  exceed their minimum required thresholds. Otherwise,  $D_k$  will apply for an empty sub-band for exclusive use.
- 2) *Sub-band Allocation:* all D2D links use Algorithm 1 introduced in Section IV-A to choose their sub-bands.
- 3) *Hierarchical Matching:* all D2D links will first use Algorithm 1 in Section IV-A to choose their sub-bands. Then, the D2D links with exclusive sub-bands can decide whether to aggregate their sub-bands with each other using Algorithm 2 proposed in Section IV-C.

Note that, as we have proved in Section IV-C, if the belief functions of all D2D links converge, the spectrum sharing structure of D2D links can always converge to a stable structure. In the rest of this section, we mainly focus on the cases that D2D links have already established their beliefs. We will discuss the convergence rate of the belief updating algorithm in (11) at the end of this section.

Let us consider a cellular system consisting of multiple D2D links (denoted as blue lines in Figure 3) and cellular subscribers (denoted as red triangles in Figure 3) that are uniformly randomly located in a square-shaped coverage area, as shown in Figure 3. To simplify our discussion, we focus on the downlink transmission and assume each D2D link consists of a source (denoted as a blue circle) and a destination (denoted as a green circle). In a practical system, D2D communication should only be enabled when the source and destination are close enough. We hence assume each destination is uniformly randomly located within a fixed radius (e.g., 20 meters in our simulation) of the corresponding source. We consider the payoff of D2D links defined in (1) - (3) and let the

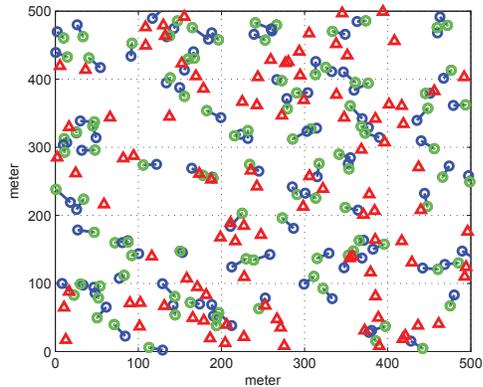


Fig. 3. Simulation setup: we use  $\triangle$  to denote cellular subscribers, blue lines to denote the D2D links, blue  $\circ$  to denote D2D transmitters and green  $\circ$  to denote D2D receivers.

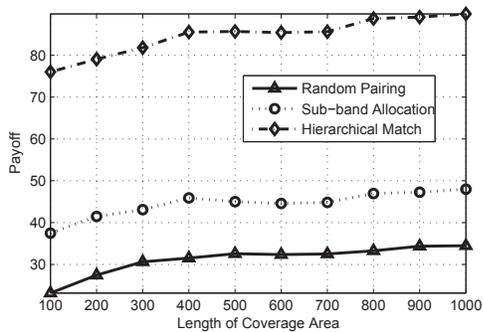


Fig. 4. The total payoff of D2D links under different coverage areas in a cellular network with 100 D2D links and 100 cellular subscribers.

channel gain between two D2D links  $D_k$  and  $D_n$  and one D2D link  $D_k$  and one cellular subscriber  $P_j^i$  be  $h_{xD_k} = \frac{\tilde{h}_{xD_k}}{\sqrt{d_{xD_k}^\sigma}}$  where  $x = D_n$  or  $P_j^i$ ,  $\tilde{h}_{xD_k}$  is the channel fading coefficient following the Rayleigh distribution,  $d_{xD_k}$  is the distance between  $x$  and  $D_k$ , and  $\sigma$  is the pathloss exponent.

In Figure 4, we fix the number of cellular subscribers and D2D links and present the total payoff of D2D links

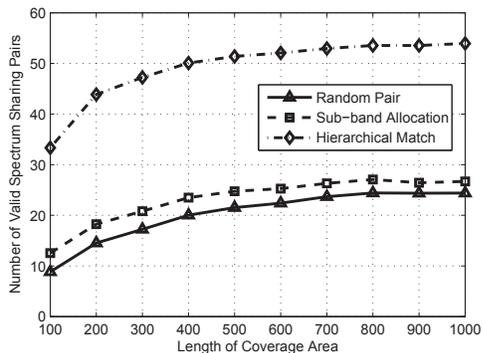


Fig. 5. The number of D2D and cellular or D2D and D2D spectrum sharing pairs for different coverage area sizes in a cellular network with 100 D2D links and 100 cellular subscribers.

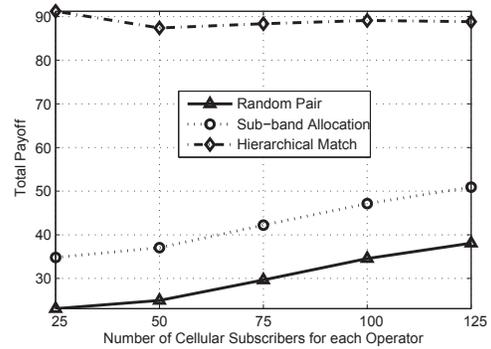


Fig. 6. The total payoff of D2D links for different number of cellular subscribers in a system with 100 D2D links.

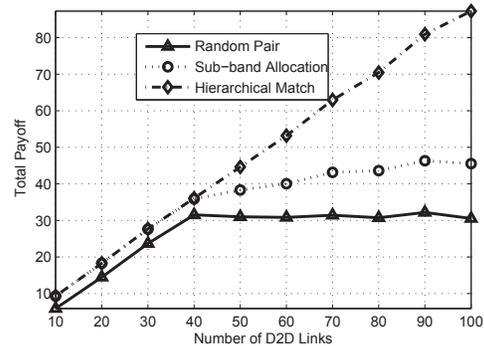


Fig. 7. The total payoff of D2D links for different number of D2D links in a system with 100 cellular subscribers.

under different lengths of the square-shaped coverage area with a range from 100 to 1000 meters. Our considered coverage area covers the femtocell, pico-cellular ( $< 200$  meters), micro-cellular ( $> 200$  meters) and macro-cellular ( $> 1000$  meters) systems [28]. We observe that the random pairing method achieves the worst payoff among all the methods. If we allow each D2D link to decide its sub-band using Algorithm 1, the payoff of each D2D link can be improved. We also observe a remarkable performance improvement by allowing spectrum sharing

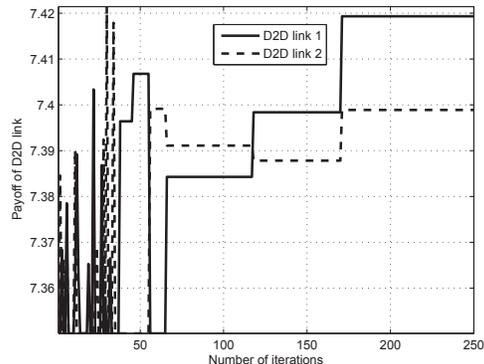


Fig. 8. The convergence rate of payoffs of two D2D links with mixed selection using the belief updating algorithm.

among D2D links with exclusive sub-bands by using Algorithm 2 proposed in Section IV-B.

To compare the spectrum sharing capacity in terms of the number of D2D links that can be supported by the existing cellular system, we present the number of valid spectrum sharing pairs formed between a D2D link and a cellular subscriber or two D2D links in Figure 5. We observe that, compared to random pairing, both sub-band allocation and hierarchical matching can almost double the spectrum sharing capacity, especially in the femtocell or pico-cell (coverage length  $< 200$  meters). This is because when the coverage area becomes small, the cross-interference between the spectrum sharing D2D links and cellular subscribers becomes critical and, in this case, choosing the cellular subscribers that are far from each D2D link becomes important to improve the spectrum sharing capacity of the systems.

We fix the number of cellular subscribers to compare the payoffs of D2D links with different numbers of cellular subscribers in Figure 6. Increasing the number of cellular subscribers provides each D2D link with more choices and hence can increase the payoffs of the D2D links with both random pairing and sub-band allocation. In addition, the payoff of the hierarchical matching increases at a faster speed than that of random pairing when the number of cellular subscribers increases.

In Figure 7, we fix the number of cellular subscribers and consider the total payoff of D2D links, varying the number of D2D links in the coverage area. We observe that if the number of D2D links is small, most of the D2D links can find cellular subscribers to share spectrum with, and hence allowing spectrum sharing between D2D links with exclusive sub-bands (i.e., hierarchical matching) cannot provide any payoff improvement. However, continuously increasing the number of D2D links provides more choices for each D2D link with an exclusive sub-band when it wants to share its sub-band with other D2D links using Algorithm 2.

The convergence of Algorithm 3 is illustrated in Figure 8, where we pick two D2D links and present their payoffs with hierarchical matching under different iterations. It can be observed that the payoffs of the chosen D2D links can converge to a stable state after the initial fluctuations during the training period under certain conditions.

## VI. CONCLUSION

In this paper, we have considered spectrum sharing between D2D links and cellular networks. We have developed a hierarchical stable matching market with incomplete information to study the D2D spectrum sharing problem. We derive a sufficient condition for which the stable spectrum sharing structure exists. We propose a distributed algorithm to detect whether the sufficient condition is satisfied and, if it is, the algorithm leads to a stable spectrum sharing structure.

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